

# Prelim

CS410, Summer 1998

17 July 1998

Please note:

- This exam is closed book, closed note. Sit every-other seat.
- Put your answers in this exam.
- The order of the questions roughly follows the course presentation and not necessarily order of difficulty.
- There are 5 questions totalling 100 points and 1 extra credit question. Point values are indicated for each question.
- Feel free to separate pages; you can re-staple them at the end of the exam.
- You have 75 minutes to complete the exam. Do not continue writing after time is up.
- Partial credit is available for all questions. However, keep your answers concise – extraneous ramblings will take your time and may hurt you.
- Answers will be posted on the web.

1. Prove formally that if  $f(n)$  is  $O(g(n))$ , then  $g(n)$  is  $\Omega(f(n))$ . [10 points]

2. (a) What form of recurrence relations can be solved using the Master Theorem? (That is, state the part of the theorem which describes the relations for which the theorem applies.) [6 points]
- (b) Write a recurrence relation that cannot be solved by the Master Theorem. (Read the next part before answering, though.) [6 points]
- (c) Write a program (pseudo-code is fine) that has a running time accurately expressed by your answer to the previous question. Be sure to explain how the variable in your recurrence relation relates to your program's input. [6 points]

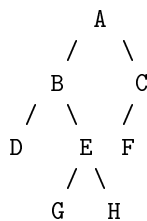
3. Below is a partial implementation for a data structure that is like a **sorted singly-linked list**. In addition, every node in the list has a pointer to the  $k^{th}$  next thing in the list.
- Assume the implementation uses sentinels, i.e., *do not worry about following null pointers*. That is, if a next or kth pointer would be null, assume it points to a node whose key is “infinity”. Equivalently, just assume you’re never “near the end of the list”.
  - Assume there are no duplicate keys.
  - You will not be graded on Java syntax. Pseudo-code is okay, but do **not** use it to avoid “low-level details”.

```
class ListNode {
    int key;
    Object val;
    ListNode next;
    ListNode kth;
}

class List {
    ListNode head;
    Object lookup(int key) {...}
    void delete(int key) {...}
    ...
}
```

Write an efficient lookup method for this data structure. Assume a node with the key exists. Your method should examine less than  $n/k + k$  nodes in the worst case. [15 points]

4. This problem concerns ways to print all the keys in a binary tree. Let the term “print a node” really mean “print the key at a node”. Assume you have a method `print` which takes a node and prints its key. The following tree serves as an example below:



- (a) Suppose we wish to print nodes before their children and left subtrees before right subtrees. (So the result for the example would be “A B D E G H C F”.) Write a non-recursive program to accomplish this task. Use an auxiliary ADT from class so that the program you write below is short. (Hints: Make sure you pick the right ADT. Your program is done when the ADT object is empty.) [12 points]
- (b) Now suppose we wish to print nodes in a top-to-bottom order with nodes at the same level printed left-to-right. (So the result for the example would be “A B C D E F G H”.) Write a non-recursive program to accomplish this task. Use an auxiliary ADT from class so that the program you write below is short. (Same hints as in previous problem) [12 points]
- (c) Your previous two answers should look very similar. Explain how could you have used object-oriented programming to avoid cut-and-paste programming. Hint: To make your explanation simpler, you may assume that the methods of your two auxiliary ADTs have the same names. [8 points]

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5. This problem concerns priority queues.

- (a) Explain how a 2-3 tree can be used to implement a priority queue. Give the asymptotic running times of the operations. [5 points]
- (b) Now consider a 2-3 tree *without the restriction that the leaves be ordered*. Explain how such a tree could be used to implement a priority queue. (Specifically, explain how the operations differ in implementation from those in the previous problem.) Give the asymptotic running times of the operations. [10 points]
- (c) To “merge” priority queues  $P_1$  and  $P_2$  means to make  $P_1$  a priority queue that has all the elements originally in  $P_1$  or  $P_2$ . Explain how priority queues implemented as 2-3 trees with **unordered** leaves could be merged. Assume  $P_1$  is bigger than  $P_2$ . Do not bother to write code for the merge method – just describe what it would do. The running time of your solution should be  $O(\log n)$  where  $n$  is the size of  $P_1$ . (Hint: How would you insert a whole tree at once?) [10 points]

**Extra Credit** [10 points]

Write an efficient delete method for the data structure in problem 3. The method takes a key, finds the node with that key, and removes that node. Assume a node with the key exists. Your method should examine less than  $n/k + 2k$  nodes in the worst case. (Hint: Notice the 2.)